

# Deriving a spatially-explicit hillslope sediment delivery ratio model based on the travel time of water across a hillslope

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**Abstract:** To derive a sediment budget at the catchment scale, sediment transport models such as SedNet typically require an estimate to be made of the input of sediment to each stream link for each defined sub-catchment. This requires an estimate of both the erosion and delivery of sediment from every pixel within each sub-catchment. The total erosion within each pixel is calculated using the Universal Soil Loss Equation (USLE), while the delivery of sediment from that pixel to the nearest stream is based on a hillslope delivery ratio (HSDR). At large scales (250 m), a constant HSDR is typically used, however at the 5 m scale used in this study, a spatially-explicit HSDR was required. To derive this, a physics-based hillslope erosion model, LISEM, was applied and calibrated to a representative, monitored hillslope in the Weany Creek catchment. The results of this modelling approach were then generalised to derive a relationship between travel time and HSDR which was then applied to the whole of the Weany Creek catchment.

**Keywords:** Hillslope sediment delivery ratio, LISEM model, SedNet model.

## 1. BACKGROUND

As catchments are heterogeneous, determining the source areas of hillslope erosion requires this heterogeneity to be accounted for. At a fine scale (5 x 5 m pixels in this paper), both the erosion within each pixel and the delivery of that sediment to the catchment mouth needs to be accounted for. This is typically done using a hillslope sediment delivery ratio (HSDR), and at this fine scale, HSDR needs to account for landscape heterogeneity through being spatially-explicit (ie varying from pixel to pixel). A companion paper to this one (Kinsey-Henderson and Post, this volume) suggests that at larger scales (30, 90, and 250 m pixels), the need for a spatially-explicit hillslope delivery ratio is reduced.

The problem of determining an appropriate sediment delivery ratio to use has been around for many years. Walling (1983) supported the use of a spatially explicit sediment delivery ratio when he stated that “each sediment source should be viewed as possessing a unique delivery potential and the probability of sediment being exported from a particular sources should be related to its

relative position with respect to the stream and the basin divide”. We have previously interpreted this statement as meaning that the sediment delivery ratio should decrease with linear distance from a stream, and this was the basis for the exponential decay function with distance from stream presented in Kinsey-Henderson *et al.* (2005b):

$$HSDR_i = 0.1366e^{(-0.0091d_i)} \quad (1)$$

where  $HSDR_i$  is the sediment delivery ratio of the  $i^{th}$  pixel and  $d_i$  is the linear distance to the nearest stream.

However, this exponential decay of HSDR with increasing distance from stream fails to take into account other important factors. The most obvious of these are slope, vegetation cover, and hydraulic versus linear paths to stream. These affect the velocity and capture (eg via infiltration) of water on the hillslope and therefore its sediment carrying capacity. One approach which takes these factors into account is the use of travel time to predict SDR – a concept proposed by Ferro and Minacapilli (1995), Ferro (1997), and Ferro *et al.*

(1998). They propose that SDR can be related to travel time by the following relationship:

$$SDR = e^{(-\gamma t)} \quad (2)$$

Where  $t$  is the median travel time within a morphological unit of a catchment and  $\gamma$  is a coefficient, considered to be a constant for a given catchment.

Jain and Kothyari (2000) applied this concept to catchment studies at a fine scale by explicitly calculating travel time for each grid point within their experimental catchments. This has the advantage of accounting for the effect of distance from stream, and changes in cover and slope along individual flow paths. The travel time for each pixel is the integration of all travel times along the flow path to the nearest stream. In Jain and Kothyari (2000), travel time for an individual pixel in a flow path is calculated as follows

$$t_i = \frac{d_i}{v_i} \quad (3)$$

where  $d_i$  is the distance of the hydraulic path to stream and  $v_i$  is the velocity of the water according to

$$v_i = a_i \sqrt{S_i} \quad (4)$$

where  $S_i$  is the slope of the hydraulic path to stream and  $a_i$  is a co-efficient related to landuse as defined by Haan *et al.* (1994) and SCS (1975).

Jain and Kothyari then apply the view of Ferro (1997) that “the delivery effects into the channel system can be neglected for small basins in which well-developed flood-plains do not exist” by defining channel pixels from which delivery is assumed unity. In effect, these channel pixels are treated as if they have travel times of 0, a view supported by the authors, at least in the case of suspended sediment from small basins. Jain and Kothyari’s study suggested average SDR’s for their experimental catchment of around 0.7 and values approaching unity near streams. These SDR’s seem high compared to those observed in small catchments in the Burdekin (Post *et al.*, 2005a).

## 2. METHODS

(2) provides an equation relating the sediment delivery ratio of a pixel to the travel time of sediment from that pixel into the nearest stream. The drawback with (2) is that it dictates that SDR must approach unity next to the streams (i.e with very short travel times). We believe that this is a

poor representation of reality in our current study for the following reasons:

1. The Universal Soil Loss Equation (USLE) provides an estimate of the *total* erosion of sediment from a pixel. This includes both coarse and fine grained sediment. As this study is only concerned with fine grained (suspended) sediment, the maximum value that SDR should be able to take is the proportion of fine compared to total sediment eroded within a pixel. As hillslope soils in this catchment consist of 36% fine and 64% coarse grained material (Post *et al.*, 2005a), the SDR should be considerably less than unity (it may however be greater than 0.36 as fine sediment tends to be preferentially eroded before coarse sediment).

2. Measurement of erosion from representative hillslopes in the Weany Creek catchment has shown that on average approximately 275 tonnes per year of fine grained sediment (silt and clay) is delivered from hillslopes (Bartley *et al.*, 2006). Application of (2) in the Weany Creek catchment produces the rather unlikely result that all of this silt and clay is sourced from the 5 x 5 m pixels immediately adjacent to the stream. That is, *no* sediment is delivered from further up the hillslope into streams.

As a result, we propose an additional term to (2) which can be interpreted as the maximum sediment delivery ratio,  $\beta$ . The relationship between travel time and SDR then becomes

$$HSDR_i = \beta e^{(-\gamma t_i)} \quad (5)$$

It will be noted that (5) is of very similar form to (1) with travel time replacing distance to stream. In Kinsey-Henderson *et al.* (2005b), based on a-priori assumptions about the way the hillslope functioned,  $\beta$  was set to 0.1366 and  $\gamma$  was set to 0.0091. The purpose of the current paper then is to derive appropriate values for  $\beta$  and  $\gamma$  based on measurements of delivery of fine sediment from hillslopes and results from the application of a physics-based model, LISEM to one of the monitored hillslopes in the Weany Creek catchment.

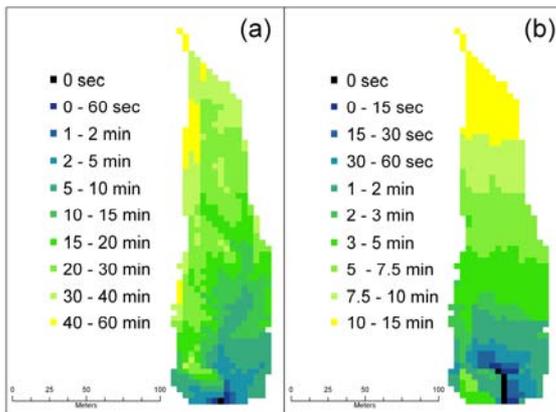
## 3. RESULTS

### 3.1 LISEM versus Ferro travel times

LISEM is a physically-based hydrological model developed in the Netherlands (Jetten, 2003). It works by routing water over terrain surfaces using a grid-based routing scheme, and as such, can be viewed as a bottom-up modelling approach as defined by Post *et al.* (2005b). Details of the

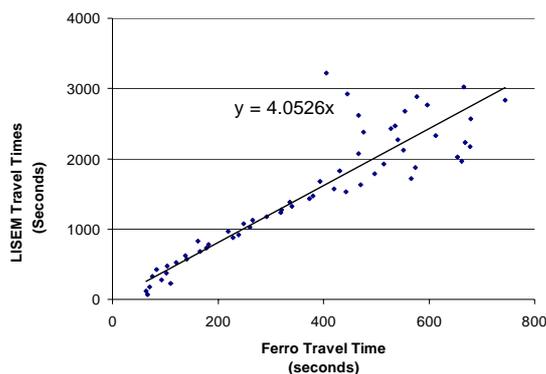
application of the LISEM model to the Flume 1 hillslope in the Weany Creek catchment can be found in Kinsey-Henderson *et al.* (2005a).

The LISEM model was used to determine the fluxes of water from each 5 x 5 m pixel on the 1.2 ha Flume 1 hillslope. Because the model routes water from pixel to pixel over this hillslope, we can use the results to estimate the travel times required for water to exit the hillslope. Travel times estimated from maximum event flow velocities are shown in Figure 1 along with those calculated by the modified version of the Ferro travel time as given by (3) and (4).



**Figure 1:** Modelled travel time from (a) LISEM and (b) Ferro for the Flume 1 hillslope

Travel times for the Flume 1 hillslope as derived by the LISEM model versus those derived using the Ferro approach ((3) and (4)) are shown in Figure 2.



**Figure 2:** Modelled travel time from LISEM versus modelled travel time from Ferro

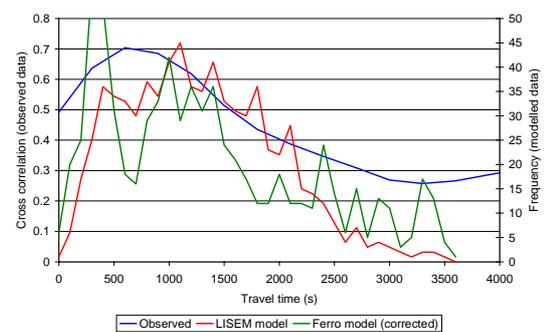
It will be seen from Figure 1 and Figure 2 that travel times calculated using the Ferro approach are approximately four times smaller than those

calculated using the LISEM model. The reasons for this are presumably related to the different techniques used, where the LISEM model routes water from cell to cell using a physics-based approach, while the Ferro approach takes a very simple, conceptual approach, relating travel time to distance travelled, slope and land cover.

However, despite the difference in the magnitude of travel times from the two approaches, there is a strong linear relationship between the travel times calculated using the two different approaches (Figure 2). The LISEM approach is too complex and requires too many data inputs to apply across the whole of the Weany Creek catchment, however, the Ferro approach is yielding travel times which appear to be around 4 times too short. As we have reason to believe that the LISEM travel times are reflecting reality (see Section 3.2 below), we have multiplied the Ferro travel times by 4.05 such that the mean Ferro travel time is equal to the mean LISEM travel time on the Flume 1 hillslope.

### 3.2 Comparison with observed travel times

As we have monitored the rainfall and runoff from the bottom of the Flume 1 hillslope shown in Figure 1, an estimate of the travel times down this hillslope can be obtained by deriving the cross-correlation between rainfall and runoff from the hillslope (Post and Jones, 2001). This cross-correlation and its comparison to the LISEM and modified Ferro travel times are shown in Figure 3.

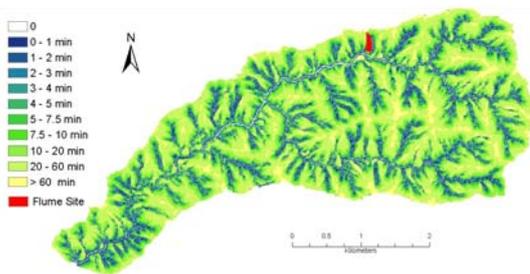


**Figure 3:** Observed, LISEM and Ferro travel times for the Flume 1 hillslope

Both the average travel time and range of possible travel times on the Flume 1 hillslope are reproduced by the LISEM and Ferro models (Figure 3). Both models display a large degree of scatter compared to the observed travel times, however the observed travel times are artificially smoothed by the cross-correlation procedure, so this is not a major concern. The reason for the large dip in the number of pixels with Ferro travel

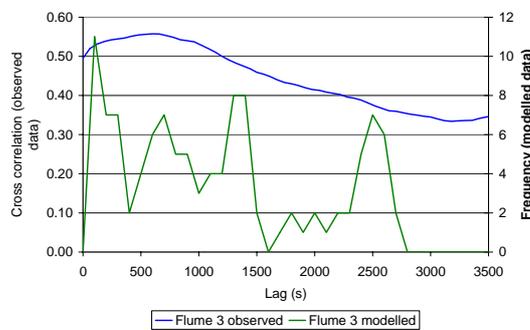
times of 600-700 seconds is not known, but is presumably related to the combination of slope, cover and distance to stream of these pixels. The increase in the cross-correlation between rainfall and streamflow seen at around 4000 seconds is interesting. It may be an artefact of the cross-correlation procedure, or it may reflect sub-surface flow on the hillslope reaching the flume. Either way, as they only model surface flow, we would not expect either of the models to reproduce this behaviour.

Application of the Ferro travel time algorithm to each 5 x 5 m pixel in the Weany Creek catchment yields the results shown in Figure 4.



**Figure 4:** Ferro travel times for the Weany Creek catchment

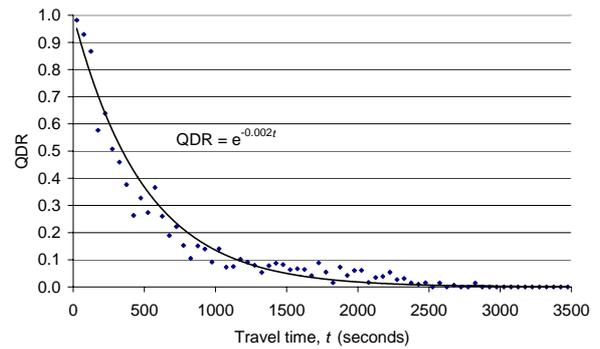
As we currently monitor two other hillslopes in the Weany Creek catchment for rainfall and discharge (Flume 2 and Flume 3, see Bartley *et al.*, 2006 for details), we can compare the modelled travel times from the Ferro approach to those from the cross-correlation between rainfall and discharge from these flumes. The comparison for Flume 3 is shown in Figure 5. While the Ferro travel times do not match the observations as well as they did on the calibration Flume 1, they do appear to be around the same size as the observed travel times. Results for Flume 2 are not shown here as subtleties of terrain led to a mismatch between the location of Flume 2 with respect to the derived flow pathways in the DEM.



**Figure 5:** Observed and Ferro travel times for the Flume 3 hillslope

### 3.3 Deriving $\gamma$ from LISEM results

As we now have the travel time of water for each pixel in the Weany Creek catchment, the next step is to derive appropriate values of  $\gamma$  and  $\beta$  to use in (5). We will use results from the LISEM model to derive  $\gamma$  as follows. The relationship between the discharge delivery ratio, QDR (the proportion of water generated by a cell which exits the hillslope), and travel time (based on the maximum event flow velocity observed in LISEM) is shown in Figure 6.



**Figure 6:** Modelled discharge delivery ratio versus modelled travel time for the Flume 1 hillslope

Based on Figure 6 we can see that a value of  $\gamma$  of 0.002 reproduces the exponential relationship between travel time and QDR. Note that the relationship shown in Figure 6 is for water and not sediment. However, this study is only concerned with the very fine fraction of sediment. This fraction can be considered to be that sediment which, once suspended, will remain in suspension as long as the water containing it continues to move. Given this caveat, we believe that using water as a surrogate for suspended sediment is justified.

### 3.4 Deriving $\beta$ from hillslope flume results

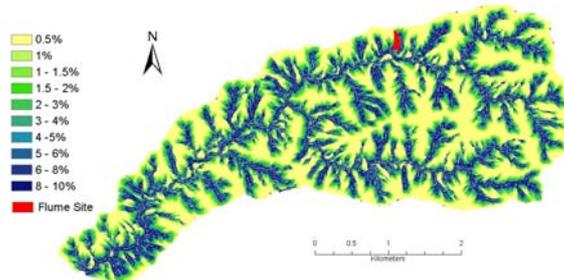
Results from the hillslope flumes in Weany Creek indicate that approximately 275 tonnes per year of suspended sediment is sourced from hillslopes (Bartley *et al.*, 2006). This represents approximately one-third of the total suspended sediment exported from the Weany Creek catchment (Post *et al.*, 2005a).

The total erosion within all of the 5 x 5 m pixels in the Weany Creek catchment has been estimated from the Universal Soil Loss Equation to be 7577 tonnes per year (Kinsey-Henderson *et al.*, 2005b).

We can use the total amount of erosion (7577 tonnes) and the total amount of delivery of fine sediments to stream in the Weany Creek catchment

(275 tonnes) to determine the appropriate value of  $\beta$  to use in (5) such that the derived values of HSDR balance the erosion of sediment with the delivery of sediment to stream. A value of 0.1 for  $\beta$  was calculated as necessary to achieve this.

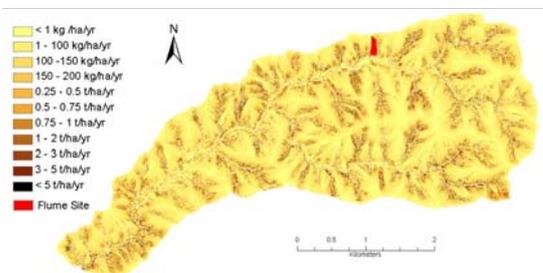
The spatially-explicit values of HSDR produced from the application of the derived  $\gamma$  and  $\beta$  values (of 0.002 and 0.1 respectively) in (5) are shown in Figure 7.



**Figure 7:** Spatially-variable hillslope delivery ratios in the Weany Creek catchment

### 3.5 Sediment yields

Having derived a spatially-explicit hillslope delivery ratio for the Weany Creek catchment (Figure 7), and having previously calculated total erosion from the USLE (Kinsey-Henderson *et al.*, 2005b), we are now able to derive a spatially-explicit representation of the sources of fine sediment which are delivered to stream in the Weany Creek catchment. This is shown in Figure 8. Both the magnitude and spatial arrangement of delivery of fine sediment to stream in Figure 8 are consistent with the patterns we would expect to see in reality, although to confirm this result through field trials is a large undertaking and is beyond the scope of the present study.



**Figure 8:** Predicted hillslope erosion of fine sediment which reaches streams in the Weany Creek catchment

## 4. SOME THOUGHTS ON BOTTOM-UP AND TOP-DOWN MODELS

The approach represented in this paper is a fairly simple one, whereby travel times determined using a detailed, physics-based model are used to help parameterise a much simpler but more widely applicable conceptual model. While this is a simple concept, we believe that it represents a possible way in which bottom-up models can be used to improve the results and applicability of top-down models.

That is, although top-down models are simple enough to be applied across a number of sites, they may not have sufficient process representation to be able to relate their parameters directly to catchment attributes (as is required if the model is to be applied to ungauged catchments). One way around this may be to gain information about catchment behaviour through the application of a more detailed bottom-up model and then transfer the process-based understanding to the top-down model.

In the current study for example, the relationship between the delivery of water from a hillslope pixel to a stream was related to the travel time of that water using a detailed bottom-up physics-based model. However rather than apply that model to the whole catchment (an impossible exercise given the data requirements), this relationship was used to parameterise a simple top-down hillslope delivery model.

## 5. CONCLUSIONS

Travel times determined using a detailed physics-based model, LISEM were used to help parameterise a simple travel time model based on slope, cover and distance from stream. The resultant model is relatively simple to apply at either a large or small catchment scale, and seems to produce feasible predictions of the spatial nature of hillslope delivery of sediment.

A companion paper, Kinsey-Henderson and Post (this volume) shows that this model can be scaled to larger pixel sizes, through the recognition that channel pixels are a mixture of hillslope and channel (a further improvement we have made on the Jain and Kothiyari (2000) approach). This is necessary, because data inputs at the scale at which the model was applied in the current paper (5 x 5 m pixels) are generally not available for most small catchments, and are not applicable at larger catchments because of the enormous datasets produced. This model will be applied to the entire Burdekin catchment (120,000 km<sup>2</sup>) in

NE Australia in order to test this approach at the larger scale.

## 6. ACKNOWLEDGEMENTS

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