# Scenario Analysis in Water Resources Management Under Data Uncertainty

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**Abstract:** In water resources management problems, uncertainty is mainly associated with the value of hydrological exogenous inflows and demand patterns. Deterministic models are inadequate to represent these problems and traditional stochastic optimization models cannot be used if there is insufficient statistical information to support the model. In this paper the uncertainty is modelled by a scenario approach in a multistage environment which includes different possible system configurations in a wide time horizon. A robust chance optimization model is used in order to obtain a so-called barycentric value with respect to decision variables. The successive reoptimization step, based on this barycentric solution, allows reducing the consequences deriving from a wrong decision. The improved version of WARGI DSS performs scenario analysis by identifying trends and essential features on which to base a robust decision policy. The current version of WARGI can be linked to commercial solvers as well as to some free solvers such as IdrScen. IdrScen is a new package for large dimension problems based on open source philosophy, that exploits the speed of network simplex methods in order to obtain very efficient solutions to the scenario problems. Moreover, the application to a real water resource system in Sardinia, Italy, shows the usefulness of the scenario analysis in water resources problems affected by a high level of uncertainty in data input. It appears that IdrScen can be a promising alternative tool to commercial codes for large size optimization problems coming for complex real resource systems.

***Keywords***: Water Resources; Uncertainty; DSS; Scenario Analysis; Pro-active approach

## 1. INTRODUCTION

Water Resources (WR) problems are typically characterised by a level of uncertainty regarding the value of data input such as supply and demand patterns. Assigning inaccurate values to them could invalidate the results of the study. Consequently, deterministic models are inadequate for the representation of these problems where the most crucial parameters are either unknown or are based on an uncertain future.

The traditional stochastic approach gives a probabilistic description of the unknown parameters on the basis of historical data. This is a very efficient approach when a substantial statistical base is available and reliable probabilistic laws can adequately describe parameters' uncertainty and their possible outcomes [Infanger, 1994; Kall and Wallace, 1994; Ruszczynski, 1997].

It is well known that stochastic optimisation approaches cannot be used when there is insufficient statistical information on data estimation to support the model, when probabilistic rules are not available, and/or when it is necessary to take into account information not derived from historical data.

In these cases, the scenario analysis technique could be an alternative approach [Dembo, 1991; Rockafellar and Wets, 1991]. Scenario analysis can model many real problems where decisions are based on an uncertain future, whose uncertainty is described by means of a set of possible future outcomes, called "scenarios". Therefore, a scenario represents a possible realisation of some sets of uncertain data in the time horizon examined.

The scenario analysis approach considers a set of statistically independent scenarios, and exploits the inner structure of their temporal evolution in order to obtain a "robust" decision policy, in the sense that the risk of wrong decisions is minimised.

Some examples are given in Escudero [2000], Pallottino et al. [2005] for water resources management, in Mulvey and Vladimirou [1989] for investment and production planning, in Glockner [1996] for air traffic management and in Hoyland and Wallace [2001] for insurance policy and production planning. In this paper he authors improve the approach already presented in Pallottino et al. [2005] and propose a reoptimization procedure that follows the scenario analysis. This approach has been developed for water systems in the EU Projects WAMME [WAMME, 2003] and SEDEMED [SEDEMED II, 2005] and applied to real cases. In the following paragraphs, it is illustrated the application of scenario analysis to a real water system in south Sardinia, Italy.

## 2. WATER RESOURCES CHANGE DYNAMIC OPTIMIZATION MODEL

In WR management problems, particularly under water scarcity, deterministic models are not adequate to describe the variability of some crucial component in the water balance between sources and demands. Even small differences in data can produce a significantly different solution and management criteria have to take account of it. In the scenario analysis for WR (Pallottino et al, 2005) each scenario represents a possible realization of some sets of uncertain data in the time horizon examined. Typically, most of the data can be affected by an uncertainty but a high level of uncertainty in WR problems is referred to exogenous inflows in water bodies and uses demand patterns. The last few years have shown just how extreme meteorological events can be, especially in Mediterranean area, and it is hard to represent these events by a probabilistic law. In an uncertain environment the stochastic optimization approach cannot be adopted since it is unreliable to match a valid occurrence probability to each scenario. One common approach is to solve a set of optimization problems for a number of generated series (parallel scenarios) followed by a simulation phase of each scenario in order to obtain a different water management policy for each scenario. All policies are completely independent one from the other because they are obtained from scenarios analysed separately. As a consequence, the decisions adopted are closely related to the scenario selected at the end of the simulation and the study must start all over if a different scenario comes true. The scenario analysis approach attempts to face the uncertainty factor by taking into account a set $\boldsymbol{G}$ of different supposed scenarios corresponding to the different possible time evolution of uncertain data.

In scenario optimization, unlike simulation, the different scenarios are considered together to obtain a global set of decision variables on the whole set of scenarios. More precisely, two scenarios sharing a common initial portion of data must be considered together and partially aggregated with the same decision variables for the aggregated part, in order to take into account the two possible evolutions in the subsequent different parts. In this way, the set of parallel scenarios is aggregated by producing a tree structure, called *scenario-tree* [Pallottino et al., 2005]. The aggregation rules guarantee that the solution in any given period is independent of the information not yet available. This result can be obtained by inserting *congruity constraints* which require that the subsets of decision variables, corresponding to the indistinguishable part of different scenarios, must be equal among themselves [Rockafellar and Wets, 1991].

The problem supported by the *scenario tree,* is described by a mathematical model that includes all single-scenario problems plus some inter-scenario linking constraints representing the requirement that if two scenarios $g1$ and $g2$ ($g1,g2 \in \boldsymbol{G}$) are identical up to time $t$ on the basis of information available at that time, then the corresponding set of decision variables, $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$, must be identical up to time $t$. These constraints represent the congruity requirement that the subsets of decision variables corresponding to the indistinguishable part of different scenarios must be equal among themselves.

Moreover, a weight can be assigned to each scenario representing the “importance” assigned by the manager to the running configuration. At times the weights can be viewed as the probability of occurrence of the examined scenario. More often they are determined on the basis of background knowledge about the system.

The resulting mathematical model is named *chance-model* to indicate that it is not stochastically based but, due to the impossibility of adopting probabilistic rules and/or to the necessity of inserting information that cannot be deduced from historical data.

A LP chance model ($\boldsymbol{P_C}$) can have the following general structure [Pallottino et al., 2005]:

$$\begin{aligned} &\min\left(\sum_g w_g c_g x_g\right) \\ &A_g x_g = b_g, \forall g \in G \qquad\qquad (1) \\ &l_g < x_g < u_g, \forall g \in G \\ &x^* \in S \end{aligned}$$

where $w_g$ represents the weight assigned to a scenario $g \in \boldsymbol{G}$; $\boldsymbol{x^*}$ represents the vector of variables submitted to congruity constraints; $\boldsymbol{x^*} \in \boldsymbol{S}$. The first two sets of constraints represent standard constraints for each scenario $g$. To generate the set $\boldsymbol{G}$ of scenarios, different approaches such as Monte Carlo generation scheme, Neural network techniques or ARMA models can be performed. The aim of this paper is not to detail these procedures and we assume that the set $\boldsymbol{G}$ is available.

Regarding weight definitions, if the manager was able to evaluate the weight $w_g$ as the probability that scenario $g$ will occur, he could estimate it by some stochastic technique or statistical test. More often the manager has few, if any, possibilities to do this due to the difficulty in deriving a probabilistic rule from statistical considerations. Instead, in scenario analysis, a weight $w_g$ assigned to a scenario $g$ can be interpreted as the "relative importance" of that scenario in the uncertain environment. In other words, in scenario analysis, weights are interpreted as subjective parameters assigned on the basis of the experience of the water management board.

The weights attribution can be crucial in case of drought period scenarios for water resource systems management. If events of water scarcity occur, a rationing policy must be adopted in time by water managers in order to avoid limitations in priority demands satisfaction. An effective management policy must be able to establish a target value in reservoirs and aquifers for delivering resources to the priority demand centres even in occurrences of water scarcity.

Nevertheless, the community suffers less from resource rationing if it has been forewarned of a possible shortage. Decision variables related to establish target values and rationing criteria can be assented taking into account the entire range of possible scenarios of resource availability, neither too pessimistic in the case abundance will occur, nor too optimistic in the case of scarcity of resources.

In other words, a target value should be sufficiently barycentric in respect to the different possible scenarios that could take place in the future. Establishing the resource demand level at this target value would permit notifying the resource users (the community) in a timely fashion. As a consequence, preventive measures could be adopted in order to avoid, at least in part, damages derived from an unexpected drastic cut in satisfaction of demands. A similar approach can be easily extended from water resources management problems under uncertainty to other types of resources management (i.e.: oil, raw materials, currency, transportation, telecommunications, etc.).

If $\hat{x}^t_g$ are the decision variables representing the resources that can be delivered to a demand centre in time-period $t$ under scenario $g$, we want to determine a target demand as the value $x^b$ that is barycentric with respect to all $\hat{x}^t_g$. To obtain this value we introduce in the objective function of problem (***P**$_C$*) a function measuring the weighted distance from $x^b$ to $\hat{x}^t_g$ for all $g$ and $t$. If we adopt the Euclidean norm to measure this distance, the chance barycentric model (***P**$_B$*) can be expressed as:

$$\begin{aligned} &\min \sum_g w_g f_g(x_g) + \sum_g \sum_t \lambda_g (\hat{x}^t_g - x^b)^2 \\ &s.t. \\ &x_g \in X_g, \qquad \forall g \in G \\ &x^* \in S \end{aligned} \tag{2}$$

where $l_g$.is the weight associated to the norm.

Once the value $x^b$ is determined, a re-optimisation process can be set in order to identify the sensitivity of the examined system with respect to deficit programming.

For WR systems considered in the paper, the reoptimization model has been constructed as a deterministic dynamic model in which the redefined demand is settled equal to the barycentric value $x^b$ . Checks on residual damages on the system have be done adopting as data input, those corresponding to the most crucial scenario (e.g. the one that the manager considers the most risky for the system). The difference between the new configuration of delivered resources in each time-period $t$ and the value $x^b$, identifies the set of *no-programmed* deficits for the system.

In the sample system illustrated in the following section we determine a value $x^b$ in such a way that it is barycentric with respect to all $x^t_g$. We then reoptimize the system solving a deterministic model assigning to the demand centre the obtained value $z^b$ as target value and adopt, as data input, those corresponding to scarce scenario.

The solution of re-optimization model give as results the resources delivered to the demand centre in the re-optimisation phase together with the *programmed* deficits (given by differences between the initial configuration of resource demands in each time-period $t$ and the barycentric value $x^b$) and the *no-programmed* deficits (difference between the original resource demand and the value $x^b$).

Moreover, comparing the behavior of delivered resources in different scenarios using barycentric values, give us the possibility to evaluate the efficiency of management policy and losses in case others scenarios will occur. The programming of deficits can be done using different level of critical states and makes it possible for the manager to set up adequate preventive measures which permit a notable reduction in losses due to resources scarcity.

## 3. A REAL PHYSICAL SYSTEM

In accordance with the Sardinia Regional Water Plan, scenario analysis was performed in collaboration with the Ente Autonomo del Flumendosa (EAF – Regional Water Board) on a real water system in south Sardinia, Italy, in different configurations. Synthetic results obtained for practical applications are shown in this paragraph. In particular, reported results are referred to the center of the system (Medio Flumendosa) that is considered one of the main pivots of the system as it can control water transfers to the principal demands. Since 1987, the Sardinia Water Plan has highlighted the necessity of defining an optimal water works assessment and optimal management rules for water system. Correct evaluation of system performances and requirements became increasingly urgent, as system managers were obliged to face serious resource deficits caused by the drought events of the past decade accompanied by an almost total uncertainty in hydrological inflows. The main water supply source is represented by three reservoirs with a total storage capacity of 584.1 million cubic meters ($Mm^3$). Gravity galleries connect the reservoirs. No significant aquifers are present in the system. Total yearly average distributed volume in the period examined is 235.2 $Mm^3$ for civil, industrial and agricultural demands. In this practical application, these types of demand are represented by 3 different centers, each characterized by the total request of civil, industrial and agriculture sector equal respectively to 115.7 $Mm^3$ , 39 $Mm^3$ and 80.5 $Mm^3$. Civil and industrial demands are constant along the year while agricultural demand is monthly variable. The simplified schematization carried out using a specialized graphical user interface WARGI-GUI [Sechi and Zuddas, 2000] for the system, is reported in Figure 1.

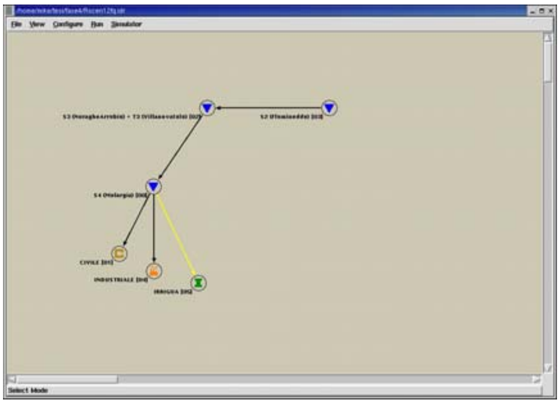

**Figure 1.** Case study in the GUI schematization

The basic hydrological data is derived from the report in [RAS et al., 2003] and different scenario generation techniques have been compared. Starting from a database with a time-horizon up to 75 years, corresponding to 900 monthly time-periods, a set of 30 scenarios was submitted to statistical validation and selected. Scenario analysis was performed on a scenario-tree of 2 and 3 stages up to 30 leaves. Since each scenario involves about 3.000 variables, the change model supports several thousands of variables and constraints. In this paper, we report some results obtained adopting a scenario tree with a time-horizon of 48 time-periods and a branching time in the 12th time period [Pallottino et al., 2005] Two scenarios are deduced from the last 4 years of hydrological inflows reported in [RAS et al., 2003]. We adopt these data as scenario g1 while scenario g2 is derived from assuming that a reduction of 50 percent will occur after branching time.

The change model can be written as follows:

$$\min \sum_{t.=1}^{48} \left[ \sum_{j=1}^{5} (c_j^t x_{j,g1}^t + c_j^t x_{j,g2}^t) + \sum_{i=1}^{3} (a_i^t u_{i,g1}^t + a_i^t u_{i,g2}^t) \right] \tag{3}$$

$$r_{r,\min}^t Y_{r,\max} < y_{r,g}^t < r_{r.\max}^t Y_{r,\min}\,, \quad t = 1,\dots,48; \qquad r \in R$$

$$y_{r,g}^{t-1} - y_{r,g}^t - x_{j,g}^t = inp_{r,g}^t$$

$$x_{j,g}^t + u_{i,g}^t = p_{j,g}^t\,, \quad i \in D$$

$$y_{r,g1}^t = y_{r,g2}^t$$

$$x_{j,g1}^t = x_{j,g2}^t\,, \quad j \in A$$

$$u_{i,g1}^t = u_{i,g2}^t$$

where R, D and A are respectively the set of reservoirs, demands and transfer arcs in the simplified system.

Objective function and constraints will be analytically expressed on the basis of the feature of the examined system. Variables of the optimization problem, for each scenario *g* at time-period *t*, are referred to stored resource ($y^t_g$), delivered resource ($x^t_g$) from reservoirs to different types of demands ($p_j$). Stored resources are bounded by lower and upper constraints in the model. Deficits $u^t_g$ represent the difference between demand *p* and delivered resources $x^t_g$, in each time-period *t*. In the objective function $c^t_j$ and $a^t_i$ represent the associated costs.

In this paper, we illustrate some comparisons between transfer water to demand centre obtained by deterministic model with independent scenarios *g1* and *g2* and optimization chance model with aggregated scenarios.

As reported in Figure 2, deterministic model defines optimal fluxes configurations where demands are fulfilled in scenario *g1* (the transferred water coincides with the demands) while under condition of water scarcity, as in scenario *g2*, during the last 12 months deficits are equal to the total demands and there isn't water availability in the reservoirs. In this application the deterministic optimization model is not conducive to incorporating risk and uncertainty in hydrological input. In our experience, the need for convincing the decision-makers in Water Authority to supplement their judgement with WARGI-DSS moves around the possibility of considering the uncertainty with model predictions of the impacts of their possible decisions. Obviously uncertainty don't make decision making easier! Incorporating realistic hydrologic uncertainty, WARGI-DSS with scenario analysis defines a configuration of drought mitigation measures that contribute to human welfare. The behavior of the flows obtained by scenario analysis shows that in the scenarios *g1* and *g2* demands are partially satisfied during the whole time horizon and high priority demands (e.g. civil demands) are not interested by heavy shortfalls. In the scenario analysis, supply reductions aim to minimize the possible drought impacts on the system. According with the deficit penalization costs associated to different types of demands, only the demands with lower priority (agricultural demands) are affected by deficits.

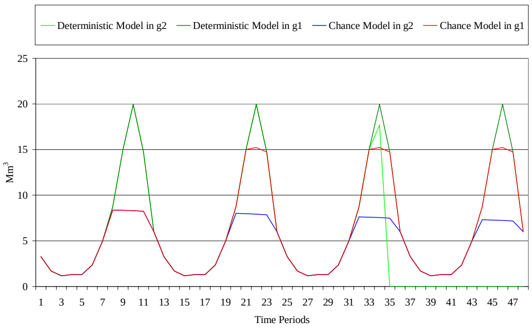

**Figure 2.** Transfer volume in deterministic and scenario models

Using the change barycentric model, we calculated the value $x^b$ that is barycentric with respect to all water transfer from reservoirs to agricultural demand. In order to make this, in the chance model *(3)* we modify the objective function as follows:

$$\min \sum_{t=1}^{48} \left[ \begin{array}{l} \sum_{j=1}^{5} \left( c_j^t x_{j,g1}^t + c_j^t x_{j,g2}^t \right) + \sum_{i=1}^{3} \left( a_i^t u_{i,g1}^t + a_i^t u_{i,g2}^t \right) + \\ + \lambda_g (x_{a,g1}^t - x^b)^2 + \lambda_g (x_{a,g2}^t - x^b) \end{array} \right]$$

where $a$ is the agricultural demand.

Measures of system performance, frequently used by Water Authorities, are reliability (how often the system fails) and vulnerability (how significant the consequences of failure may be). Table 1 illustrates the values of these indicators obtained by deterministic optimization, already used by Sardinian Water Authority, and chance model.

**Table 1.** System Reliability and Vulnerability with different optimization models.

| | Deterministic Model | Chance Model |
|---|---|---|
| Temporal Reliability | 71% | 94% |
| Vulnerability | 94% | 27% |

We determined a barycentric value equal to 62.88 Mm$^3$ (Figure 3). The reduction in agricultural demand satisfaction is equal to 80.5-62.88=17.62 Mm$^3$ and it's designed as programmed deficit. This approach could be very useful in agricultural where the economic consequences for water deficiencies are different according to different temporal horizon of predicted demand. The new value of 62.88 Mm$^3$ is a long run demand that is the value of demand water during the planning period in which the farm operator decides or not to keep the land under farming while the old value of 80.5 Mm$^3$ is a short run demand that is the value of irrigation water based on water applied within a single irrigation season, after crop have already been planted [Sulis, 2006].

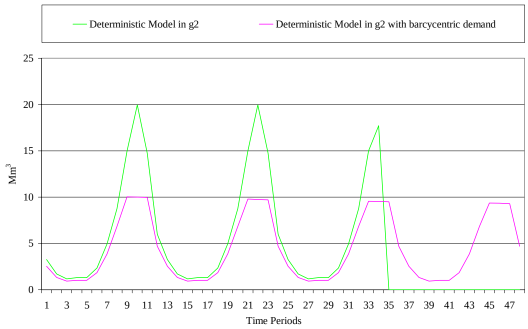

**Figure 3.** Transfer Volume in deterministic model using or not barycentric value

In order to obtain a robust decision policy that minimize the economic consequences of possible drought events, the Water Authority could use the barycentric value from scenario optimization and update the agricultural demand configuration.

## 4. RESOLUTION TECHNIQUES: AN OPEN SOURCE APPROACH

Open source software and free software are terms used to describe approaches and philosophies under which certain computer software is made available to the public. Open Source environment provides an opportunity for scientific community and practitioners to benefit, update and develop software, sharing ideas and experiences with people dealing with the same interest in order to continuously expand the common knowledge and improve the efficiency of the computer codes. The proposed scenario analysis tool is embedded into the DSS named WARGI (WAter Resource system optimization aided by Graphical Interface). WARGI is a Open Source software developed by the University of Cagliari (Italy) and composed by several independent macro-modules implemented in C++ and Tcl/Tk. The results presented in this paper are obtained interfacing WARGI with Cplex, a commercial solver for linear and quadratic optimization problems. WARGI can be linked to any mathematical programming solver in order to benefit by the most efficient state-of-the-art computer codes in the field. Models describing the water resource planning and management optimization problems under uncertainty conditions, show a special structure which suggests some specialized approaches in order to overcome the serious computational problems due to their very large dimension. Specialized algorithms can be adopted to solve this kind of problems having a set of constraints simple to deal with and a set of complicating constraints. Lagrangian relaxation and decomposition techniques can be adopted to solve the equivalent deterministic problem supported by the scenario tree. In scenario analysis the complicating constraints are represented by requirements on interperiod transfers (non-anticipativity constraints). These constraints are relaxed, that is moved, and added to the objective function as a penalisation factor. As a result, the remaining set of constraints, exhibits a block diagonal structure that can be split up coming to solve a set of reduced sub-problems. Bundle technique collects the sub-solutions and produces the overall solution or, at least, a good approximation of it. We implemented this approach in an Open Source tool named IdrScen [Manca, 2006] that can deal with real problems under data uncertainty. The presence of uncertainty has effect in the dimension of the problem, as the number of scenario grows, the full problem could become hard to solve with standard algorithms and existing open source software such as Lp_solve.

## 5. CONCLUSIONS

In this paper we show a practical application of scenario analysis in a real water system in South Sardinia (Italy). It appear that this approach can be very useful in order to decide a set of planning and operational measures when the system is affected by a high level of uncertainty in supply or demand patterns. The reoptimization deterministic analysis uses the barycentric value from a previous scenario optimization and defines a robust decision policy that minimizes the risk of wrong decisions. The proposed scenario analysis tool is imbedded into a Open Source DSS, named WARGI, that can be linked with commercial or free solvers. At the moment, we implemented the bundle technique in an Open Source tool named IdrScen useful to solve huge problems under data uncertainty.

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