Real Time Optimal Resource Allocation in Natural Hazard Management

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Abstract: In the emergency management phase relevant to the occurrence of a catastrophic natural event, the efficiency of the emergency system can be deeply influenced by a correct assignment of the available resources to the "demand" centers, i.e., those elements of the territory that are directly involved in the event. In this paper, a general formulation of the real time optimal resource allocation problem is presented, and it is formalized as a mathematical programming problem. The dynamics relevant both to the resources over the territory and to the prediction of the behavior of the natural phenomenon are suitably taken into account. A graph model is introduced in order to describe the territory under consideration, and resources can be located either in the nodes or are in transit over the direct links. The phenomenon dynamics can be associated to the nodes on the territory, and specifically to the demand centers. The general approach has then been applied to the forest fire hazard. A simple model aiming to describe the spread of a fire over the territory has been developed, and it has been used for the formulation of the mathematical programming problem. A specific case study, relevant to the forest fire hazard in Liguria region (Italy), has been here defined and tested, and the main results are briefly reported and discussed.

***Keywords**:* Decision support systems, real time natural hazard management, environmental risk, forest fires, resource allocation problems.

1. INTRODUCTION

When a catastrophic natural event occurs, the efficiency of the emergency system can be deeply influenced by a correct allocation of the available resources to the "demand" centers, i.e., those elements of the territory that are directly involved in the event.

The optimal allocation of resources is a well-known problem in Operation Research, which has been faced following many approaches. In the field of catastrophic event management, some specific approaches can be found: for example, in the resource management module of the TRACE system an allocation procedure is present to define an allocation plan [Paggio *et al.*, 1999], while the work by Friedrich *et al.* [2000] is relevant to the optimal resource allocation after an earthquake. However, a modern view of civil protection command centers [Wybo, 1992] would require an integrated approach of the several natural risks, which may occur on a territory, with a clear formulation of the cost/benefit of a resource allocation plan. In this connection, strategic importance is assumed by the operative phase, when the event is occurring or about to occur; it is worth mentioning that no specific approach can be found in the literature as regards this issue.

The main goal of this work is to define the structure of a decision support system (DSS), to aid decision makers in the optimal allocation of the resources required to manage an emergency due to a catastrophic natural event. The main module of the DSS is represented by a mathematical programming problem, whose general formulation is provided in this paper, in order to provide a common framework to treat all kinds of natural hazards. However, each kind of natural hazard requires a special formulation of the dynamics of the event and of the resources that are necessary to cope with the emergency. A case study is described in this work, referring to the real-time management of forest fire hazard.

2. THE PROPOSED APPROACH

In a resource allocation problem, first of all, the following issues are to be classified:

- the representation of resources either via continuous or via integer variables;
- the service demand that may be distributed or concentrated over the territory.

As a matter of fact, every alternative in both issues could be a reasonable modeling approach in the emergency management of natural hazards: however, for the sake of brevity, an exhaustive analysis of such issues is left to forthcoming papers.

In this work, which is oriented towards a forest fire management hazard application, it seemed reasonable to choose a continuous representation for the resources, and a concentrated one for the service demand; these features are shown in the next section.

A general formalization for the real time resource allocation problem, with continuous resources and concentrated demand, is so hereinafter introduced. Then, the specific problem relevant to forest fires hazard is considered, and a detailed formulation of the resulting mathematical programming problem is provided.

2.1 The general formalization

It is assumed that both demand centers and resource location centers are represented as nodes of the directed graph G(V,L), where V is the set of nodes, and L is the set of the links among those nodes.

If j is a generic node belonging to V, let us indicate by $P(j)$ (resp. $S(j)$) the set of nodes predecessor (successor) of node j.

The general problem formalization refers to a time horizon (of a suitable length) of T time intervals. It is assumed that each link belonging to the set L is characterized by a unitary transit time, that is the time required by the resources in order to transit over the link. This assumption can be generalized to include links with transit time greater than one, by introducing a suitable number of dummy nodes, each one characterized by null service demand.

The primary decision variables of the problem are:

$U_j(t)$: amount of resources assigned to node j during time interval (t, t+1), $t=0,\ldots,T-1$, $j\in V$;

$w_{jl}(t)$: amount of resources that during time interval (t, t+1), $t=0,\ldots,T-1$, move from node j to node l.

Further variables necessary for the formalization of the problem are:

$D_j(t)$ service demand in j during time interval (t, t+1), $t=0,\ldots,T-1$ (it may result from dynamic model whose behaviour is influenced by the amount of resources assigned to that particular node).

Besides, the parameters $\tilde{U}_j$ indicate the amount of resources located at each node of the considered graph at the initial time instant of the optimization horizon.

The cost function of the proposed general formulation is composed by three terms: the first one is relevant to the estimated damage, the second one is relevant to the inadequate assignment of resources to the nodes of the graph, and the last one relevant to the transfer cost between the nodes.

The first term has to be written in order to penalize the total amount of estimated damage in each node and in each time interval, and can be expressed in the general form

$$P_j(t) = f_j^P\left(D_j(t)\right).$$

The second term is introduced in order to take into account the possible partial incompatibility among the resources and a location center (e.g., the base may be not well equipped for a particular kind of resources); the functions expressing the cost of inadequate assignment of resources to nodes $O_j(t)$ can be expressed in the general form

$$O_j(t) = f_j^O\left(U_j(t)\right).$$

Finally, the transfer cost $s_{jl}(t)$ between nodes is introduced in order to penalize the resource movements among the nodes. Such a cost takes into account the total amount of resources moving on each link for each time interval, and can be expressed as:

$$s_{jl}(t) = f_{jl}^s\left(w_{jl}(t)\right).$$

The general formalization of the dynamic (real-time) resource allocation problem is the following

$$\min \sum_{t=0}^{T-1} \sum_{j\in V} f_j^P\left(D_j(t)\right) + \sum_{t=0}^{T-1} \sum_{j\in V} f_j^O\left(U_j(t)\right) +$$

$$+ \sum_{t=0}^{T-1} \sum_{j\in V} \sum_{l\in S(j)} f_{jl}^s\left(w_{jl}(t)\right) \qquad (1)$$

s.t.

$$D_j(t+1) = f_j^D\left[D_j(t), U_j(t)\right] \qquad j\in V$$

$$t = 0,\ldots,T-1 \qquad (2)$$

$$U_j(0) = \tilde{U}_j - \sum_{l\in S(j)} w_{jl}(0) \qquad j\in V \qquad (3)$$

$$U_j(t+1) = U_j(t) - \sum_{l\in S(j)} w_{jl}(t+1) + \sum_{l\in P(j)} w_{lj}(t)$$

$$j\in V \quad t = 0,\ldots,T-1 \qquad (4)$$

$$\sum_{j\in V} U_j(t) + \sum_{j\in V} \sum_{l\in S(j)} w_{jl}(t) - \sum_{j\in V} \tilde{U}_j = 0$$

$$t = 0,\ldots,T-1 \qquad (5)$$

$$U_j(t) \geq 0 \quad j\in V \qquad t = 0,\ldots,T-1 \qquad (6)$$

$$w_{jl}(t) \geq 0 \quad j \in V \quad l \in S(j) \; t = 0,...,T-1 \quad (7)$$

As a matter of fact, natural hazard management is in general characterized by two different dynamics: the first one is relevant to the physical process that characterizes the specific event (flood, forest fires, etc.), whereas the second one is relevant to the resources intervention. The two different dynamics nay be mutually related; for example considering an earthquake phenomenon, one can observe that after the physical event, the main goal of the resources is that of rescue the maximum number of injured people as early as possible; besides, the survival possibility of such people could be characterized by a dynamics strongly correlated with the dynamics of the intervention resources. In case of forest fire, as it will be described in the next section, risk is related with the fire spread dynamics, which is strongly correlated with the fire attack provided by the resources able to face the propagation of the fire.

Coming back to the above formalization, it is worth observing explicitly that: constraints (2) are relevant to the dynamics of the process for a single node; constraints (3) and (4) are related to resource kinematics; constraints (5) are related to the conservation of resources over the territory; constraints (6) and (7) ensure the non-negativity of the decision variables.

2.2 Application of the general formalization to the forest fire hazard management

During real time management of forest fires, one of the main objectives is to position the firefight resources in an optimal way to face the ongoing fire events, taking into account their dynamics.

The territory is modeled as a graph with three types of nodes: fires, fire fight stations and transit nodes, introduced to satisfy the hypothesis that the transfer time on all arcs is unitary.

The notation used in the formulation of this problem is analogous to the one introduced in the previous section. Furthermore, let us define $Y \subset V$ as the subset of nodes belonging to V and representing fires. The (continuous) resources are represented by the amount of extinguishing power, and variables $U_j(t)$, $w_{jl}(t)$ are expressed in kW.

The demand $D_j(t)$ - which plays a key role both in the objective function and in the constraints representing the process dynamics in a real time resource allocation problem - is supposed to be simply the overall power which characterizes the fire corresponding to a given node, namely

$$D_j(t) = p_j(t) \quad (8)$$

In the past years, several approaches aiming at modeling forest fires dynamics in space and time have been proposed in the literature. Among the others, the models belonging to the so-called semi-physical class seem to be the most practical and widely used by technicians and the scientific community. Semi-physical models take into account the physical laws governing the fire spread phenomenon by means of parametrical relationships among fuel characteristics, meteorological variables and topography. The parameters of the model can be estimated on the basis of empirical observations consisting of in-field measurements, related to real case studies, or to experimental fires, or collected in a combustion laboratory (Rothermel, 1972; Van Wagner, 1977; Albini, 1985).

However, as the purpose of this work is that of evaluating the effectiveness of the proposed approach in connection with simple case studies, and since the discussion and the selection of suitable propagation models are far beyond the scope of the present work, only a very simple fire propagation model will be used.

Such a model is characterized by a perfect isotropy (absence of orographic asperities, absence of wind, etc.). In this case, a rough approximation is to consider the rate of increase of the burnt area as linearly dependent on the overall power of the fire, and the rate of increase of the power itself as constant (at least, as far as no extinguishing action is taken). The equations representing such quite simple dynamics are

$$\begin{cases} \dfrac{da_j(t)}{dt} = k_1^j \cdot p_j(t) \\ \dfrac{dp_j(t)}{dt} = k_2^j \cdot 1(t) \end{cases} \qquad j \in Y \qquad (9)$$

where

$a_j(t)$ $[m^2]$	is the area burnt by fire j at instant t;
$p_j(t)$ $[kW]$	is the power of fire j at instant t.

In this simple representation, if the fire front is modeled as a circumference, a linear increase (with time) of the radius $r_j(t)$ gives a quadratic increase of the area $a_j(t)$.

Assuming a linear intensity p_{lin} $[kW/m]$ of the fire, constant over the circumference, the overall power $p_j(t)$ of the fire can be expressed as

$$p_j(t) = p_{lin} 2\pi r_j(t) \quad (10)$$

Thus, the first equation of (9) can be re-written as

$$da_j(t) = 2\pi r_j(t) dr_j = k_1^j p_{lin} 2\pi r_j(t) dt \quad (11)$$

that allows obtaining

$$k_1^j = \frac{dr_j}{dt} \cdot \frac{1}{p_{lin}} \quad (12)$$

where

$\frac{dr_j}{dt}$ is the spread speed of the fire.
From the second equation of (9)

$$dp_j = k_2^j dt \tag{13}$$

Then, differentiating equation (10) and substituting in equation (13), one obtains:

$$p_{lin} 2\pi dr_j = k_2^j dt \tag{14}$$

so that k_2^j is equal to

$$k_2^j = \frac{dr_j}{dt} \cdot 2\pi \cdot p_{lin} \tag{15}$$

The discretization of (9) provides

$$\begin{cases} a_j(t+1) = a_j(t) + k_1^j p_j(t)\Delta t \\ p_j(t+1) = p_j(t) + k_2^j \Delta t - U_j(t) \end{cases} \quad j \in Y \tag{16}$$

where it has been taken into account the action of the extinguishing resources $U_j(t)$.

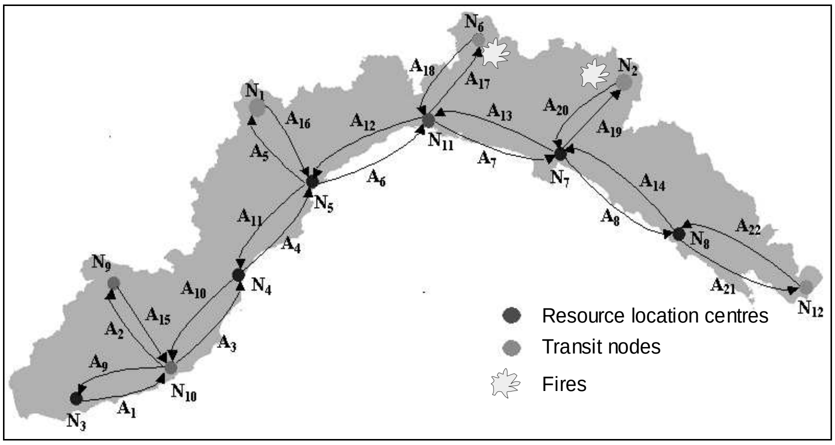

Figure 1. Representation of the target area and of the directed graph used in order to formalize the optimal real time resource allocation problem relevant to forest fires risk in the Liguria region. The two fires located near nodes 2 and 6 are the ones relevant to Scenario 2.

As regards the functions needed to describe the real time resource allocation problem objective, $f_j^o(\cdot)$ has been omitted since it is not relevant for the considered case, because only one kind of resources has been considered. Specifically, only the amount of water that can be carried with truck engines is considered. Furthermore, such kind of resources can indifferently be assigned to each one of the considered location centers. Thus, no inadequate location for the resources can be found among the considered nodes (namely, nodes corresponding to location centers, fires, or transit nodes).
Besides, it is assumed

$$P_j(t) = \gamma_{j,t} a_j^2(t) \tag{17}$$

$$s_{jl}(t) = \eta_{j,l,t} w_{jl}^2(t) \tag{18}$$

where $\gamma_{j,t}$ and $\eta_{j,l,t}$ are suitable weight parameters.
Then the overall problem can be stated as

$$\min \sum_{t=0}^{T-1} \sum_{j \in Y} \gamma_{j,t} a_j^2(t) + \sum_{t=0}^{T-1} \sum_{j \in V} \sum_{\substack{l \in V \\ l \neq j}} \eta_{j,l,t} w_{jl}^2(t) \tag{19}$$

subject to constraints (3) ÷ (7), (16).
Note that the value of quantities $a_j(0)$ for each $j \in Y$, and $U_j(0)$, for each $j \in V$, are supposed to be known.

3. APPLICATION TO TEST CASES

The model described in the previous section has been applied to the Liguria region. Among the Italian northern regions, Liguria is the one that is more affected by this kind of calamity, with more than 500 fires a year and a very high rate between the total burnt area (about 3060 ha in 2002, and 4560 in 2003) and the total forest area (about 334000 ha).
Forty-four Forest Service's stations are spread over the territory, with an overall nominal value of extinguishing power of about 270000 kW. This value has been obtained by considering only the available land-force resources placed in each station and under the simplifying assumption that the water flow given by the considered mean reaches completely the fire front and instantaneously evaporates [Fiorucci *et al.*, 2002].
Due to the short distance between some pairs of stations, and in order to simplify the statement and the structure of the real time resource allocation problem, a clustering of the stations in 6 nodes has

been performed. The overall graph of the considered problem is thus composed of 12 nodes (6 of them are transit nodes) and 22 bi-directional links (see Figure 1). The power present in each cluster is the result of the sum of all the power relevant to the stations of the cluster.

Two different scenarios have been taken into account, and are briefly described below.

Scenario 1 is characterized by a single forest fire, in the eastern part of the region, in correspondence of node N2; in the following, this fire will be denoted as S1F1.

Two different fires characterize scenario 2, one in the neighborhood of node N2 and one near N6 (denoted in the following as S2F1 and S2F2, respectively). The two fires are characterized by different values of parameters k^j_1 and k^j_2; in particular, S2F2 is characterized by a value of k^j_2 greater than S2F1 and, therefore it is characterized by a faster increase of the power over time.

The main characteristics of the three scenarios are resumed in table 1.

With reference to the two scenarios, two strategies are described in the following subsections: heuristic strategy, and the strategy defined by the solution of the real time resource allocation problem.

Both the scenarios are characterized by an optimization horizon T of 15 hours, subdivided into regular time intervals, equals to 1800 seconds.

	S1F1	**S2F1**	**S2F2**
dr/dt [m/s]	0.02	0.01	0.01
p_{lin} [kW/m]	1000	1000	1200
p(0) [kW]	125600	125600	150720
a(0) [m^2]	1256	1256	1256
k_1 [(m/s)/ (kW/m)]	$2*10^{-5}$	10^{-5}	$8*10^{-6}$
k_2 [(m/s)* (kW/m)]	125	62.8	73.36
Node	N2	N6	N2

Table 1. The main parameters characterizing the three fires of the two scenarios.

3.1 Heuristic strategies

In this case, the management problem is governed by a greedy heuristic strategy: the resources are allocated to the nearest fire until the demand is satisfied; if two or more fires have to be contemporarily extinguished, available resources are assigned proportionally to coefficient k^j_2.; resources can be moved from a fire and sent to another one only when an active fire is completely extinguished. Recall that the evolution of the dynamics of the fire is described by equations (9).

The single fire in Scenario 1 (S1F1) required 24 time intervals to be extinguished, affecting an area greater than 4,000,000 m^2.

Referring to Scenario 2, the fire affecting node 6 (S2F2) did not give rise to considerable damages (less than 35,000 m^2 of burned area) as all the resources were initially concentrated on this fire, because of its value of parameter k^j_2 higher than S2F1. This choice influenced the trend of fire S2F1: as a few resources were initially assigned to this fire, it reached a maximum power similar to the one of S1F1 (characterized by a higher value of coefficient k^j_2), and the burned area was greater than 1,200,000 m^2.

3.2 Real time optimal strategies (RTO)

In this case, the management problem is governed by the strategy defined by means of the solution of the optimization problem described in subsection 2.2.

S1F1 required 19 time intervals to be extinguished by the use of these strategies, whereas the burned area is equal to 250,000 m^2.

Here again, in scenario 2 resources are first assigned to S2F2, and thus this fire was extinguished before S2F1: the intervention ended in time interval 21 on S2F2, and in time interval 27 on S2F1, and the burned areas were 63,000 m^2 and 50,000 m^2, respectively.

3.3 Comparison of the strategies

It is reasonable to expect that the application of RTO strategies allows an improvement, with respect to the application of heuristic strategies, especially as regards the overall burned area. In fact, this factor is the main term of the cost function, of the optimization problem.

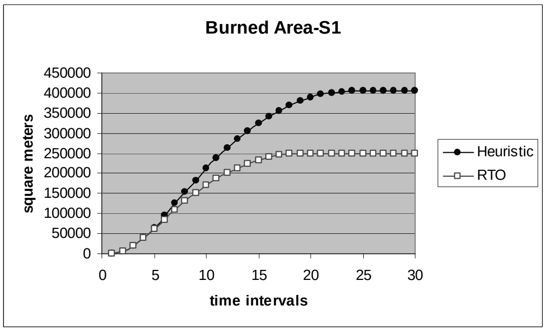

Figure 2. Cumulated burned area [m^2] relevant to Scenario 1 obtained by the use of heuristic strategies and RTO strategies.

Referring to Scenario 1 (see figure 2), it can be noted that this expectation is perfectly confirmed. In fact, a comparison between heuristic and RTO strategies shows a decrease of the overall burned area of 37%.

In order to compare the results obtained in connection to Scenario 2, it is necessary to consider jointly the fires in each scenario (i.e., S2F1 and S2F2 for Scenario 2). In fact, the cost function used for the definition of RTO strategies penalizes the global burned area, and not the burned area relevant to each fire.

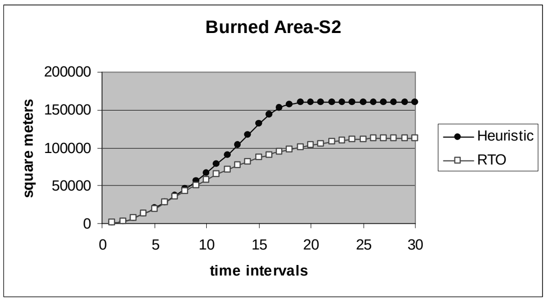

Figure 3. Cumulated burnt area [m^2] relevant to Scenario 2 obtained by the use of heuristic strategies and RTO strategies on both S2F1 and S2F2 fires.

Referring to Scenario 2, figure 3 shows that the application of RTO strategies allows a decrease of burned area of nearly 30% with respect to the results obtained by the application of the heuristic procedure.

4. CONCLUSIONS AND FUTURE DEVELOPMENTS

In the modern view of civil protection command centers, a continuous coordinated work is requested to face emergencies. In general, preventive actions before the event and coordination optimal actions either during the emergence or its very preceding instants are crucial to minimize the effects of probable catastrophes. That is why an emerging area of research is the study of methodologies and technologies, exploiting the application of interdisciplinary competences (e.g. system sciences, information technologies, operation research, telematics etc..) to support decision within this context.

In this work, the formalization of this latter aspect, specifically the real time optimal resource allocation in natural hazard management, has been presented, focusing on a case study relevant to forest fire hazard. The formalization of the decision problem, although may be subject to several improvements (for example, in the forest fire model), has the capability to support the decision makers in managing complex situations, where few resources are available and many demand centers are requesting them. In addition, the developed decision problem can be used as a training support in simulated case studies.

A future development of the proposed approach, at which the authors are already working, is the coupling of the proposed technique with another tool which is in charge to move resources in advance, when probable future requests are predicted by simulation models relevant to the specific considered hazard.

5. REFERENCES

Albini F.A., A model for fire spread in wildland fuels by radiation, *Combustion Science and Technology*, 42, 229-258, 1985.

Fiorucci P., Gaetani F., Minciardi R., Dynamic models for preventive management and real time control of forest fires, proceedings of 15th IFAC world congress in automatic control, Barcelona, Spain, 2002.

Friedrich F., Gehbauer F., Rickers U., Optimized resource allocation for emergency response after earthquake disasters, *Safety Science*, 35, 41-57, 2000.

Paggio R., Agre G., Dichev C., Umann G., Rozman T., Batachia L., Stocchero M., A cost-effective programmable environment for developing environmental decision support systems, *Environmental Modelling & Software*, 14, 367-382, 1999.

Rothermel R.C., A mathematical model for predicting fire spread in wildland fuels. USDA, Forest Service Research Paper. INT-114. Intermountain Forest and Range Experiment Station, Ogden, UT, 40 p, 1972.

Van Wagner C.E.. Conditions for the start and spread of crown fire, *Canadian Journal of Forest Research*,. 7, 23-34, 1977.

Wybo J. L.. “EXPERTGRAPH: Knowledge based analysis and real-time monitoring of spatial, application to forest fire prevention in French Riviera, proceedings of International Emergency Management and Engineering Conference, Managing Risk with Computer Simulation, Orlando, FL, 1992.