

Conceptually-based multivariate simulation of monthly runoff

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Abstract: This paper presents a multivariate extension of a parsimonious conceptually-based Auto Regressive-Moving Average (ARMA) stochastic model for monthly runoff. The multi-station model is a Contemporaneous-ARMA (CARMA), which considers separately the serial and space correlation of runoff. Serial correlation is reproduced in individual series by an ARMA model. The ARMA model residuals are uncorrelated in time but correlated in space. Spatial correlation of runoff is then reproduced by generating correlated series of residuals and using them to generate runoff through the individual ARMA models. In the conceptual framework, stochastic ARMA parameters are related to the parameters of a linear system, which represents the watershed filter that produces runoff. The system input is the effective rainfall, which is inversely estimated through the ARMA model residual. Application of the CARMA model in the conceptual framework consists in reproducing the spatial correlation on the effective rainfall rather than on the residuals. A suitable technique is also proposed for estimation of correlation in matrices with gaps. The performances of the model are discussed with regard to its application on a 9-station system in Southern Italy.

Keywords: Monthly Runoff; Conceptual model; multivariate; CARMA

1. INTRODUCTION

Simulation of simultaneous runoff series over several stations is an essential part of current practice of planning and management of water resources systems. For reasons of parsimony, it is nowadays widely accepted that multi-station runoff simulation can be achieved with models that preserve spatial correlation among stations (multi-site) and let serial correlation be reproduced by at-site univariate models (*Contemporaneous ARMA – CARMA* - models, [Salas *et al.*, 1980]). This representation means that runoff $d_{i,t}$ in station i at time t , depends on past values measured at the same station ($d_{i,t-1}$; $d_{i,t-2}$, etc.) but not on the other station's past values, as in the complete (vector) multivariate formulation. The (spatial) dependence of $d_{i,t}$ on the values occurred in the other stations is thus evaluated only on *contemporaneous* runoff values ($d_{i+1,t}$, $d_{i+2,t}$, etc.) by means of a correlation matrix of the residuals of the individual serial correlation models. This formulation reduces greatly the number of parameters to estimate with respect to the vector model, without significant loss of information. In addition, univariate models for serial correlation can be different from one station to another. This property has not yet been fully exploited in the literature, where the cumbersome tools for parameter estimation have left little room

for deeper studies on the identification phase [see e.g. Salas *et al.*, 1985].

To date, a full-featured contemporaneous periodic-ARMA model for monthly runoff is available [Rasmussen *et al.*, 1996], in which serial and spatial correlations are reproduced season by season thanks to a large parameter set. In most practical cases, however, insufficient data prevents one to use the above, powerful but demanding, model. On the other hand, conceptually-derived models [Salas and Obeysekera, 1992; Claps *et al.*, 1993] provide physical-like bases for supporting parsimonious model identification and estimation. They also present features allowing one to validate results of models applied on short or discontinuous time series. These approaches have been so far proposed only in the univariate form.

Conceptually-based models can help to overcome the following shortcomings of empirical models: i) reliance on long, continuous and contemporaneous runoff records; ii) sensitivity to normality of residuals; iii) uncertainties in identification of individual models for serial correlation. In this paper it will be shown how a CARMA model structure, that fits into the conceptually-based ARMA framework proposed by Claps *et al.* [1993], addresses the issues above.

