

SelOpt: Selection of Options based on the Balance and Ranking Method

G. Strassert

Institute of Regional Science, University of Karlsruhe, Germany, D-76128 Karlsruhe (Gustras@aol.com)

Abstract: A new multiple criteria decision-making (MCDM) method, called the Balancing and Ranking Method, is presented. The method overcomes some of the deficiencies of other MCDM methods, such as subjective evaluation of criteria weights, scoring of options, statistical estimation of weights and specification of the utility function for criteria. The new method uses a three-step procedure to derive an overall complete final order of options. First, an outranking matrix is derived, which indicates the frequency with which one option is superior to all other options based on each criterion. Second, the outranking matrix is triangularized to obtain an implicit pre-ordering or provisional order of options. Third, the provisional order of options is subjected to various screening and balancing operations that require sequential application of a balancing principle to the so-called advantages-disadvantages table that combines the criteria with the pair-wise comparisons of options.

Keywords: Multiple criteria decision-making; Balancing and Ranking Method; Triangularized outranking matrix; Advantages-disadvantages table; Overall ranking of options

1. INTRODUCTION

The new variant of MCDM presented here uses a stepwise ordering procedure to derive a transitive overall final order of a finite set of options. The term “alternatives” is avoided here because we define an alternative as a pair of options. Therefore, it would be confusing, as it is generally the case, to denominate an option an alternative. Basic features of the approach are the pair-wise comparisons of options, mixed scales and the so-called balancing principle, i.e., the balancing of vectors of advantages and disadvantages. On the other hand, explicit information on criteria weights, scoring of options or specification of a utility function for criteria is not required. The new approach intends to derive, from lessons of social choice theory, a different systematic position, already suggested by Arrow and Raynaud [1986] (see also Rapoport, 1988; Strassert, 1995; 1997

and 2000; Lansdowne, 1997; Strassert and Prato, 2002). The proposed introduction of a balancing principle leads to an integrated approach where for any pair of options the relative advantages and disadvantages are balanced and, simultaneously, the different importance of the scores is taken into account.

Following, in principle, the methodological outlines of Arrow and Raynaud [1986], the balancing principle is introduced leading to a markedly different stepwise multicriteria decision making. The example used, is for sake of illustration hypothetical and reduced in size. For a practical application see Strassert and Prato [2002].

2. DATA TABLE, OUTRANKING MATRIX AND PROVISIONAL ORDER OF OPTIONS

Each MCDM problem begins with a data table, the basic components of which are:

- (1) X is a set of options designated as P_j ($j = 1, \dots, m$).
- (2) C is a set of criteria, designated as C_i ($i = 1, \dots, n$), including definitions of scales and measures and, if necessary, a maximization or minimization postulate.
- (3) $E = [e_{ij}]$ is a set of data with $(C) \times (X)$ scores, e_{ij} . For each criterion C_i , there are j measurement results, e_{ij} . For example, e_{1m} represents the score for criterion 1 (C_1) with respect to option m (P_m).
The scales and measures can differ across criteria.

Figure 1 shows a scheme of a data table.

| Criteria | | | Options | | |
|----------|---------|-----|----------|-----|----------|
| Feature | Measure | | P_1 | ... | P_m |
| C_1 | ... | ... | e_{11} | ... | e_{1m} |
| . | ... | ... | ... | ... | ... |
| . | ... | ... | ... | ... | ... |
| C_n | ... | ... | e_{n1} | ... | e_{nm} |

Figure 1. Scheme of a data table

Each criterion C_i yields, by means of its scores, e_{ij} , an individual ranking of the options. If, for example $e_{11} > e_{12}$, the individual ranking of criterion C_1 is $C_1: \langle P_1, P_2 \rangle$.

Rankings of options normally vary by criterion.

When there is more than one unique ranking, it is necessary to derive an overall final order of options. For example, there are the following four rankings for four criteria:

$C_1: \langle P_1, P_4, P_2, P_3 \rangle$

$C_2: \langle P_4, P_1, P_3, P_2 \rangle$

$C_3: \langle P_2, P_4, P_3, P_1 \rangle$

$C_4: \langle P_4, P_1, P_2, P_3 \rangle$

Pair-wise comparisons of P_i and P_j yield an „outranking matrix,“ R [Lansdowne, 1997]. (Figure 2).

| | P_1 | P_2 | P_3 | P_4 |
|-------|-------|-------|-------|-------|
| P_1 | * | 3 | 3 | 1 |
| P_2 | 1 | * | 3 | 2 |
| P_3 | 1 | 1 | * | 0 |
| P_4 | 3 | 2 | 4 | * |

Figure 2. Outranking matrix R

The overall ranking problem with an outranking matrix is equivalent to collaborative decision-making in which each member ranks the options (candidates), and the individual rankings are used to count out votes (pros and cons). For example, (Figure 2) shows that option 1 outranks option 2 three times and option 2 outranks option 1 once, hence, option 1 receives three favourable votes and one unfavourable vote compared to option 2. The entries along the main diagonal are irrelevant in our context because they compare an option to itself.

The entries in the outranking matrix are denominated as r_{jk} ($j, k = 1, \dots, m$). If ties are not present in the ranking for any criterion, the outranking matrix satisfies the „constant sum“ property: $r_{jk} + r_{kj} = K$ for any pair of indices (j, k) (Lansdowne, 1997). In our example, without tie, K equals 4 for all pairs of indices.

Next, a triangularization procedure is applied to the outranking matrix in order to obtain a new order of options, namely: $\langle P_4, P_1, P_2, P_3 \rangle$. The resulting triangular outranking matrix, denoted by R^T , is shown in Figure 3.

| | P_4 | P_1 | P_2 | P_3 |
|-------|-------|-------|-------|-------|
| P_4 | * | 3 | 3 | 4 |
| P_1 | 1 | * | 3 | 3 |
| P_2 | 1 | 1 | * | 3 |
| P_3 | 0 | 1 | 1 | * |

Figure 3. Triangularized outranking matrix R^T

The triangular matrix systematically reorders the j options such that out of a set of $p = j!$ orders (in our case, $p = 4! = 24$), the sum of the values above the main diagonal is a maximum in the matrix of the final order. The triangularization method is generally applicable to quadratic matrices, such as

an input-output matrix or a voting matrix [Bartnick, 1991]. Triangularization has a long tradition in the context of economic input-output analysis. A state-of-the-art review is given by Wessels [1981].

In a completely triangular matrix, there are only zeros below the main diagonal, a situation which Roubens and Vincke [1985] call „total order structure“. When the latter occurs, there is a (strong) transitive overall final order of options [Banks et al., 1991; Kern and Nida-Rümelin, 1994; Roubens and Vincke, 1985; Laslier, 1997]. Normally, the order of options implied by the outranking matrix is not the final overall order of options. Therefore, triangularization can be understood as a method to both test and display the degree of achievement of a (strong) transitive overall order of options.

The degree of linearity in a triangularized matrix is measured by λ , where $\lambda = \sum_{j < k} |r_{jk}| / \sum_{j \neq k} |r_{jk}|$ and $0,5 \leq \lambda \leq 1$ [Bartnick, 1991]. The degree of linearity of the matrix given in Figure 3 is $\lambda = 19/24 = 0,79$. λ indicates how much an order of options deviates from the ideal of, in the best case, $\lambda = 1$, which implies a strong linear order, for which the transitivity condition applies (if $\langle P_i, P_j \rangle$ and $\langle P_j, P_k \rangle$, then $\langle P_i, P_k \rangle$). In the worst case, $\lambda = 0,5$, there is not a linear order, but a cycle, say $\langle P_i, P_j, P_k, P_i \rangle$, and vice versa.

In what follows, the order of options implied by the triangular outranking matrix is considered a provisional order, which is subjected to a particular screening and balancing operation. Roubens and Vincke [1985] call a similar matrix an „opinion tableau“. In this context, the outranking matrix is related to the majority rule of counting votes. It is essential for the methodological approach presented here to change the assumption of the majority rule in the context of the balancing principle. In a broader sense, the balancing principle is a particular application of the unanimity rule.

3. THE ADVANTAGES-DISADVANTAGES TABLE AND THE CORRESPONDING SET OF BALANCING PROBLEMS

Now, a new table, that is the „advantages-disadvantages table“ (Figure 4), is introduced which combines the criteria with the pair-wise comparisons of options.

The head row contains all possible pairs of options. If there are n options, the maximum number of pairs is $z = n(n - 1)/2$.

| | P ₁ /P ₂ | P ₁ /P ₃ | P ₁ /P ₄ | P ₂ /P ₃ | P ₂ /P ₄ | P ₃ /P ₄ |
|------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|
| C ₁ | | 1/3A ₁ | 1/4A ₁ | 2/3A ₁ | 2/4D ₁ | 3/4D ₁ |
| C ₂ | 1/2A ₁ | | | | | |
| C ₃ | 1/2A ₂ | 1/3A ₂ | 1/4D ₂ | 2/3D ₂ | 2/4D ₂ | 3/4D ₂ |
| C ₄ | 1/2D ₃ | 1/3D ₃ | 1/4D ₃ | 2/3A ₃ | 2/4A ₃ | 2/3D ₃ |
| C ₄ | 1/2A ₄ | 1/3A ₄ | 1/4D ₄ | 2/3A ₄ | 2/4D ₄ | 3/4D ₄ |
| ∑ A _j | 3 | 3 | 1 | 3 | 1 | 0 |
| ∑ D _j | 1 | 1 | 3 | 1 | 3 | 4 |

Figure 4. An advantages-disadvantages table for four options and four criteria

To determine the advantages and disadvantages, the data table must be processed as follows: For each pair of options the respective scores, e_{ij} , of the criteria in the head column are compared. For example, if the pair P₁/P₂ is considered, with respect to the four criteria (C₁, C₂, C₃, C₄), then we have four pair-wise comparisons of the scores, e_{ij} : e_{11} compared with e_{12} , e_{21} compared with e_{22} , e_{31} compared with e_{32} and e_{41} compared with e_{42} . These comparisons can be made independently of the scales on which the scores, e_{ij} , are represented. Hence, the pair-wise comparisons refer to quantities (cardinal scale), rankings (ordinal scale) or frequencies (nominal scale).

If, for example, e_{11} , compared with e_{12} , is superior to e_{12} (say, because comparing the cardinal scores shows that *card* $e_{11} >$ *card* e_{12}), then, option P₁, compared with option P₂, has a comparative advantage, denominated as $1/2A_1$. With respect to the second criterion (C₂), the result is, for example, a comparative disadvantage (say, because comparing the ordinal scores shows that *ord* $e_{21} >$ *ord* e_{22}), denominated as $1/2D_2$, and so on.

In the trunk of the advantages-disadvantages table appear the „votes“ of the outranking matrix (Figure 4). Obviously, the number of the positive „votes“ (pros) correspond to the number of advantages, and the number of negative „votes“ (cons) correspond to the number of disadvantages. From this point of view, one can say, that the advantages-disadvantages table shows how the quasi-votes split by criteria, or, in other words, which criteria have determined the advantages or disadvantages. Moreover, it can be said that the advantages-disadvantages table makes explicit what causes the cycles of a weighed graph: there are criteria which place option A before option B,

and there are criteria which place option B before option A.

Each column of the advantages-disadvantages table represents a separate binary decision problem. For

example, in the first column the question is if the three advantages $[_{1/2}A_1;_{1/2}A_2;_{1/2}A_4]$, taken as a bundle, dominate or do not dominate the disadvantage $_{1/2}D_3$. The answer can only be Yes or No. If the answer is Yes, then P_1 is strictly superior to P_2 , if the answer is No, then P_2 is strictly superior to P_1 , respectively.

Formally, the advantages-disadvantages table yields a set of balancing problems (in our case six) that can be written according to the following scheme:

[1] P_1/P_2 : $[_{1/2}A_1;_{1/2}A_3;_{1/2}A_4]$ are superior to $[_{1/2}D_3]$?

Yes: P_1 is strictly superior to P_2

No: P_2 is strictly superior to P_1

.

[5] P_2/P_4 : $[_{2/4}A_3]$ is superior to $[_{2/4}D_1;_{2/4}D_2;_{2/4}D_4]$?

Yes: P_2 is strictly superior to P_4

No: P_4 is strictly superior to P_2 .

[6] The sixth balancing problems, that is P_3/P_4 , is already "solved" because both the outranking matrix (Figure 3) and the advantages-disadvantages table (Figure 4) reveal one partial strict superiority relation (hidden in the data table, Figure 1).

A partial strict superiority occurs when one option shows, compared to all others options, only advantages or disadvantages with respect to all criteria. This is the case with options P_3 and P_4 , where P_3 records four disadvantages and, therefore, P_4 is strictly superior to P_3 . When this occurs, the partial strict superiority must be reflected in the final overall order of options. Hence, in the overall final order of options, whatever it is, P_4 is ranked higher than P_3 .

4. SOLVING PROCEDURE FOR BALANCING PROBLEMS

A stepwise procedure is described for solving the balancing problems. The maximum number of pair-wise comparisons and balancing problems is $z = m(m-1)/2$, which increases rapidly with m . For example, with 10 options, z equals 45. Hence, the determination of order relations is a cumbersome task, not to mention the number of criteria and the

corresponding number of advantages and disadvantages. Fortunately, it is often not necessary to make all possible pair-wise comparisons and solve the corresponding balancing problems (see below). The stepwise procedure uses the logical implications of the transitivity condition. For example, if the $m-1$ pair-wise comparisons above and alongside the diagonal are given in Figure 3, the remaining pair-wise comparisons in the upper triangle are implied by transitivity.

The principal question is how to get a maximum number of transitivity implications. This is the case when the $m-1$ pairs of options above and alongside the diagonal are given. The triangular outranking matrix given in Figure 3 indicates the following provisional ordering of options: $\langle P_4, P_1, P_2, P_3 \rangle$. It is the starting matrix used in the stepwise procedure. The goal of the stepwise procedure is to convert as many pairs of entries above the diagonal to 4 : 0 pairs as warranted by the judgements of the decision-maker. A final solution (overall final order of options) is reached when this conversion is complete. For example, if the three pair-wise comparisons above and alongside the diagonal, that is P_4/P_1 , P_1/P_2 , and P_2/P_3 , are given, the remaining three pair-wise comparisons, that is P_4/P_2 , P_4/P_3 and P_1/P_3 , are implied.

This procedure greatly simplifies the solution of balancing problems. In the best case, mentioned above, where all pair-wise comparisons above and alongside the diagonal are confirmed, only three balancing problems have to be solved.

As compared with the best case, more steps are needed to reach the final triangular outranking matrix and, hence, the final order of options. For example, suppose that the first decision is " P_1 is superior to P_4 ", instead of " P_4 is superior to P_1 ". Then, entries 4 vs. 1 also have to be inversed. This change requires new triangularization, which results in another provisional order of options and another first provisional triangular outranking matrix. Consequently, the pair-wise comparisons above and alongside the diagonal will be different (at least partly) and possibly another second balancing problem will be chosen, and so on. In the worst case, which is unlikely to occur, all (six) balancing problems have to be solved.

In complex balancing problems, the decision-maker requires help. An auxiliary table provides such support by dividing a balancing problem into partial balancing problems that are solved in a stepwise fashion. Such a table is defined as the Cartesian product of all combinations of advantages $(C_r^a, r = 1, 2, \dots, R)$ and

disadvantages (C_s^d , $s = 1, 2, \dots, S$). Using a combination operator for the option pair P_j/P_k ($j \neq k$), gives ${}^j C_r^a = 2^r - 1$ for the number of advantages and ${}^j C_s^d = 2^s - 1$ for the number of disadvantages. Colerus [1989] and Strassert [1995] give an explanation of these operators. For example, the auxiliary table for balancing problem 1 (P_1/P_2 , Figure 4) has seven ($2^3 - 1$) combinations of advantages and only one ($2^1 - 1$) “combination” of disadvantages.

5. ROLE OF JUDGEMENT AND POSTULATES OF DECISION THEORY

The above procedure differs from the traditional MCDM method of assigning a priori weights to criteria. The balancing procedure introduced allows integration of both the balancing of the relative advantages and disadvantages of (pairs of) options and, simultaneously, the taking into account the different importance of criteria. For example, when a set of advantages is not considered superior to a certain disadvantage then this result depends not only on the size but also the attributed importance of a criterion. Moreover, facing a concrete and special balancing problem a decision-maker is required to be more aware of the relative importance of criteria as compared with a situation of still poorer and general information about the decision problem when he is asked for (constant) weights in an early phase of MCDM. Confusion of the normative and factual level is a typical pitfall in this context [Prato, 1999]. The advantages-disadvantages table operates at the factual level because nothing more is presented than factual relations between the alternatives comprising each pair of options. In order to establish order relations based on the balancing principle, it is necessary to introduce the judgements of the decision-maker. While the new method avoids the need to assign weights to criteria, it does require the decision-maker to make judgements regarding the superiority of one option versus another based on their advantages and disadvantages.

Our approach corresponds to a fundamental reformulation of axioms of consumer choice theory as proposed by Gowdy and Mayumi [2001] which follows psychologists understanding cognitions as a constructive process depending on time, place, and immediate past experience. As “individual preferences for a particular item may vary considerably depending on context”, the

neoclassical axioms of invariance of preferences and non-satiation are no longer valid in the balancing and ranking method: the balancing principle paves the way for “reference dependent preferences” and takes into account that preferences are embedded in specific social and environmental contexts. Moreover, in accordance with Gowdy and Mayumi, the non-satiation postulate is without importance within the balancing and ranking method because the decision maker is free to take into account the biophysical context, in particular, the functional properties of ecosystems and their inherent set of services together with critical loads and saturation effects. And last but not least, the decision maker is not bound to the substitution postulate, since the need to assign (constant) weights to criteria is avoided, but is enabled to introduce individual ideas of complementarities of measurements results of criteria, lexicographic preferences and a hierarchy of wants.

All in all, and in mathematical terms, the balancing and ranking method focuses on the comparison of vectors (represented by the options in a data table) and abandons the transformation of vectors into scalars (as common denominators in terms of utility).

6. FINAL ORDERING OF OPTIONS

One way to view the derivation of the final order of options is as a process that eliminates from the complete enumeration of possible orders those orders that are inconsistent with the superiority relations indicated by the solutions to the balancing problems. The number of possible orders with, for example four options, is $p = n! = 4! = 24$.

Of these 24 orders, 12 orders having P_2 ahead P_4 are eliminated because balancing problem [6] (column 6, Figure 4) shows a strict superiority of P_4 to P_3 as mentioned above. Therefore, all 12 orders having P_3 ahead P_4 are disqualified.

If the pair-wise comparisons above and alongside the diagonal, P_4/P_1 , P_1/P_2 , and P_2/P_3 are, as for ease of illustration assumed, a stepwise reduction of the remaining 12 orders is as follows. Specifically, another 4 orders are eliminated from the decision $\langle P_4, P_1 \rangle$, another 5 orders are eliminated from $\langle P_1, P_2 \rangle$, and another 2 orders are eliminated from $\langle P_2, P_3 \rangle$, leaving only one order. The resulting final overall order of options is: $\langle P_4, P_1, P_2, P_3 \rangle$.

7. CONCLUSION

complete or partial final order of options. The method entails three steps. In the first step, an outranking matrix is derived from the criteria values for all options. This matrix indicates the frequency with which one option is ranked higher than the other options. In the second step, an implicit pre-ordering or provisional ordering of options is established by triangularizing the outranking matrix. The outranking matrix indicates the degree to which there is a complete overall order of options. In the third step, the provisional ordering is subjected to various screening and balancing operations based on information given in an advantages-disadvantages table. The latter indicates whether one option is superior (advantage) or inferior (disadvantage) to another option based on each criterion. The balancing problems are simplified by developing auxiliary tables that allow the decision-maker to balance the advantages and disadvantages of each pair of options until a partial or complete strict ordering of options is obtained.

8. REFERENCES

- Banks, J. S., Bordes, G. and Le Breton, M., Covering relations, closest orderings and hamiltonian bypaths in tournaments, *Social Choice and Welfare*, 8: 355-363, 1991.
- Arrow, K. J. and Raynaud, H., *Social Choice and Multicriterion Decision-Making*, Cambridge, Mass., 1986.
- Bartnick, J., Optimal triangulation of a matrix and a measure of interdependence for a linear econometric equation system. In: J. Gruber (ed.), *Econometric Decision Models*, 487-495, Berlin 1991.
- Colerus, E., *Vom Einmaleins zum Integral*. Wien, 1989.
- Gowdy, J.M. and Mayumi, K., Reformulation the foundations of consumer choice theory and environmental valuation, *Ecological Economics*, 39: 223-237, 2001.
- Lansdowne, Z. F., Outranking methods for multicriterion decision making: Arrow's and Raynaud's conjecture, *Social Choice and Welfare*, 14:125-128, 1997.
- Laslier, J.-F., *Tournament Solutions and Majority Voting*, *Studies in Economic Theory*, 7, Berlin, 1997.
- Kern, L. and Nida-Rümelin, J., *Logik kollektiver Entscheidungen*. München and Wien, 1994.
- A new MCDM method is presented called the Balancing and Ranking Method. The method uses a three-step procedure to derive an overall
- Prato, T., Multiple Attribute Decision Analysis for Ecosystem Management, *Ecological Economics*, 30: 207-222, 1999.
- Rapoport, A., *Allgemeine Systemtheorie*. Darmstadt, Germany, 1988.
- Roubens, M. and Vincke, Ph., *Superiority Modelling. Lecture Notes in Economics and Mathematical Systems*, 250, Berlin, 1985.
- Strassert, G., *Das Abwägungsproblem bei multi-kriteriellen Entscheidungen. Grundlagen und Lösungsansatz unter besonderer Berücksichtigung der Regionalplanung*, Frankfurt am Main, 1995.
- Strassert, G., *Decisión Multicriterio en la Planificación Regional: Propuesta de una nueva Metodología*, *Revista Interamericana de Planificación*, Vol. XXIX, 113: 48-66, 1997.
- Strassert, G., *The Balancing Principle, Strict Superiority Relations, and a Transitive Overall Final Order of Options*. University of Karlsruhe, Institute of Regional Science, Discussion paper no 34, Karlsruhe, Germany, 2000.
- Strassert, G. and Prato, T.: *Selecting Farming Systems using a New Multiple Criteria Decision Model: The Balancing and Ranking Method*, *Ecological Economics*, 40: 269-277, 2002.
- Wessels, H., *Triangulation und Blocktriangulation von Input-Output-Tabellen und ihre Bedeutung*. Deutsches Institut für Wirtschaftsforschung, Beiträge zur Strukturpolitik, Heft 63, Berlin, 1981.